NAME: KEY

Relative Motion Problems

One Dimensional Problems

G = ground.

1. Imagine you are a State Trooper (T) driving down the highway at 65 mph. A stupid (A) driver passes you. Their velocity relative to you is 15 mph.
   a. What is the speed of the stupid driver relative to the ground?

$V_{TG} = 65$ mph

$V_{AT} = 15$ mph

$V_{AG} = V_{AT} + V_{TG} = 15 + 65 = \boxed{80 \text{ mph}}$

   b. What is your velocity relative to the stupid driver?

$V_{TA} = -V_{AT} = \boxed{-15 \text{ mph}}$

   c. An even stupider (B) driver then goes past the OG stupid driver with a velocity of 10 mph, relative to the OG. What is the velocity of the even stupider driver relative to you?

$V_{BA} = 10$ mph

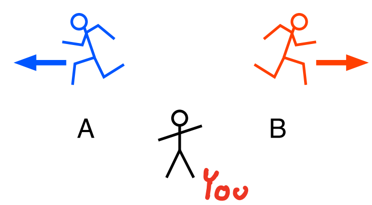
$V_{BT} = V_{BA} + V_{AT} = 10 + 15 = \boxed{25 \text{ mph}}$

   d. What is the speed of the even stupider driver relative to the ground?

$V_{BG} = V_{BT} + V_{TG} = 25 + 65 = \boxed{90 \text{ mph}}$

or $V_{BG} = V_{BA} + V_{AG} = 10 + 80 = 90$ mph :)

2. You are watching two friends, A and B, race in opposite directions. The two friends have a relative speed of 4 m/s. According to you, friend A has twice the speed of friend B.



   a. What is the velocity of A with respect to B?

$V_{AB} = -4$ m/s

   b. What is the velocity of B with respect to A?

$V_{BA} = +4$ m/s

$V_{BY} = V_{AY} + 4$

or

$V_{AY} = V_{BY} - 4$

   c. What are the velocities of the two friends relative to you?

$|V_{AY}| = |2V_{BY}|$

$\therefore V_{AY} = -2V_{BY}$ (opposite directions)

So $-2V_{BY} = V_{BY} - 4$

$3V_{BY} = 4$

$\boxed{V_{BY} = 1.33 \text{ m/s}}$

So $V_{AY} = 1.33 - 4$

$\boxed{V_{AY} = -2.67 \text{ m/s}}$

3. A bored child at an airport is playing on a moving walkway. First, they stand on the walkway, which takes them from point A to point B, a distance of 50 meters in 35 seconds. Then they run back to A, still on the walkway, in a time of 120 seconds. How fast were they running with respect to the walkway?

50 m, 35 s

W = walkway

C = child

G = ground

$V_{WG} = \frac{d}{t} = \frac{50}{35}$

$V_{WG} = 1.43$ m/s

$V_{GW} = -1.43$ m/s

Going back:

$V_{CG} = \frac{-50}{120} = -0.417$ m/s

$V_{CW} = V_{CG} + V_{GW} = -.417 + (-1.43)$

$V_{CW} = -1.85$ m/s

So $\boxed{|V_{CW}| = 1.85 \text{ m/s}}$

Relative Motion Problems

4. Object A has a constant velocity of 20 m/s according to C. The moment A passes Object B, B has a constant acceleration of 3 m/s^2 and is initially at rest, both according to C.

   a. What is the equation that gives the position of A relative to B?

   $r_{AC} = 20t \qquad r_{BC} = \frac{1}{2}(3)t^2 \qquad r_{AB} = r_{AC} + r_{CB} = \boxed{(20t) + (-1.5t^2)}$

   $\rightarrow \therefore r_{CB} = -1.5t^2$

   b. What is the equation that gives the velocity of A relative to B?

   $V_{AB} = \frac{d}{dt}(r_{AB}) = \boxed{20 - 3t}$

   c. At some point, B will catch up to A. Before then, what is the maximum separation between A and B?

   Max separation happens when relative speed is zero

   $0 = 20 - 3t$

   $t = \frac{20}{3}$

   $r_{AB} = 20(20/3) - (1.5)(20/3)^2$

   $\boxed{r_{AB} = 66.7 \text{ m}}$

   d. What is the velocity of A with respect to B when B finally catches up to A?

   $r_{AB} = 0 = 20t - 1.5t^2$

   $20 = 1.5t$

   $t = 13.33$

   $V_{AB} = 20 - 3t$

   $= 20 - 3(13.33)$

   $\boxed{V_{AB} = -20 \text{ m/s}}$

Two Dimensional Problems

5. A swimmer can swim with a speed of 5 m/s in a pool (that is her speed with respect to the water.) This same swimmer is now at a river, which has a current flowing to the East with a constant speed of 3 m/s (that is the velocity of the water with respect to the ground.) Assuming her water speed is always 5 m/s,

   $V_{SW} = 5$

   $\vec{V}_{WG} = 3\hat{i}$

   a. What would be her resultant velocity if she tries to swim due east, with the current? (This would be her velocity relative to someone on the riverbank waiting for her.)

   So $\vec{V}_{SW} = 5\hat{i}$

   $\vec{V}_{SG} = \vec{V}_{SW} + \vec{V}_{WG} = 5\hat{i} + 3\hat{i} = \boxed{8\hat{i} \text{ m/s}}$ (i.e. 8 m/s East)

   b. What would be her resultant velocity of she tries to swim due West, against the current?

   So $V_{SW} = -5\hat{i}$

   So $\vec{V}_{SG} = -5\hat{i} + 3\hat{i} = \boxed{-2\hat{i} \text{ m/s}}$ (i.e. 2 m/s West)

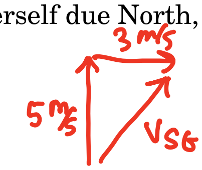
   c. What would be her resultant velocity if she points herself due North, straight across the river?

   So $V_{SW} = 5\hat{j}$

   $\vec{V}_{SG} = \vec{V}_{SW} + \vec{V}_{WG}$

   $= 5\hat{j} + 3\hat{i}$

   $V_{SG} = 3\hat{i} + 5\hat{j}$ m/s

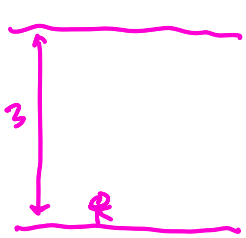
   d. If the river is 30 meters wide, how long will it take her to reach the other side of the river, if she points herself due North?

   $\vec{r}_{SG} = (3t)\hat{i} + (5t)\hat{j}$ (constant v)

   So $y = 5t$

   $30 = 5t$

   $\boxed{t = 6 \text{ s}}$

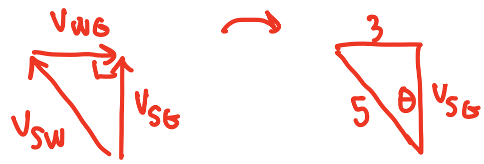


Relative Motion Problems

e. How far down stream will she be?

So $x = 3t$ $x = (3)(6) = \boxed{18\text{ m}}$

f. In what direction must she point herself so that her resultant velocity is due North, straight across the river?



$\sin\theta = \frac{3}{5}$

$\theta = 36.9^\circ$

∴ direction = 126.9° or 36.9° W of N

g. How fast is she going with this velocity (from part f)?

$5^2 = 3^2 + (v_{SG})^2$ $\boxed{V_{SG} = 4\text{ m/s}}$

h. If the river is 30 meters wide, how long will it take her to cross the river (from part f)?

$\vec{V}_{SG} = 0\hat{i} + 4\hat{j}$

So $y = 4t \rightarrow 30 = 4t$ $\boxed{t = 7.5\text{ s}}$

i. What must she do if she wanted to cross the river in the *least* amount of time? What is that time?

Must maximize velocity across the river – so ignore the river. This is actually questions c-e above

So $t_{min} = \boxed{6\text{ s}}$

6. The pilot of a plane points her airplane due South and flies with an airspeed of 100 m/s. However, there is a steady wind blowing due West with a constant speed of 40 m/s.

a. What is the resultant velocity of the airplane? (This would be the plane's velocity with respect to the ground.)

P = plane
A = air
G = ground

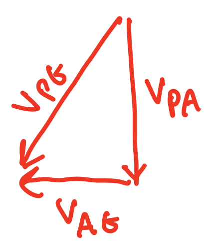
$\vec{V}_{PA} = -100\hat{j}$

$\vec{V}_{AG} = -40\hat{i}$

$\vec{V}_{PG} = \vec{V}_{PA} + \vec{V}_{AG}$

$\vec{V}_{PG} = -100\hat{j} - 40\hat{i}$

$\boxed{\vec{V}_{PG} = -40\hat{i} - 100\hat{j}\text{ m/s}}$



b. After one hour, where is the airplane in relation to its starting point?

$\Delta\vec{r}_{PG} = \vec{V}_{PG}\,t$

1 hr

$= (-40\hat{i} - 100\hat{j})(3600)$

$\boxed{\Delta\vec{r}_{PG} = -144{,}000\,\hat{i} - 360{,}000\,\hat{j}\text{ m}}$

generic unit vector notation

Relative Motion Problems

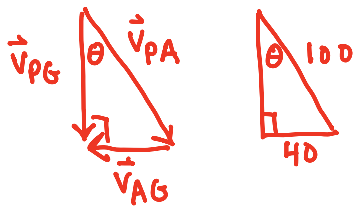
c. If the pilot wanted the plane to fly due south with respect to the ground, in what direction should she point the plane? (Assume the speed of the plane with respect to the air is still 100 m/s.)

Now

$\vec{V}_{PG} = 0\hat{i} + ?\hat{j}$

$\vec{V}_{AG} = -40\hat{i}$

$\vec{V}_{PG} = \vec{V}_{PA} + \vec{V}_{AG}$



$\sin\theta = \frac{40}{100}$

$\theta = 23.6°$

23.6° E of N (or −66.4°)

OR: $(0\hat{i} + v\hat{j}) = (100\cos\phi\,\hat{i} + 100\sin\phi\,\hat{j}) + (-40\hat{i} + 0\hat{j})$

$\hat{i}$) $0 = 100\cos\phi - 40$

$\cos\phi = \frac{40}{100} \rightarrow \phi = 66.4°$

Sin $\sin\phi$ must be negative

we need $\boxed{\phi = -66.4°}$

THE PICTURE IS MUCH EASIER & CLEANER !!

d. Using your answer from part c, how fast is the plane moving with respect to the ground?

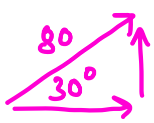
$v_{PG}^2 + 40^2 = 100^2$

$v_{PG}^2 = 8,400$

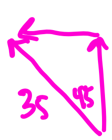
$\boxed{v_{PG} = 91.7 \text{ m/s}}$

7. A plane has a velocity of 80 m/s at an angle of 30° (North of East) with respect to the ground. The plane is flying in a wind with a velocity of 35 m/s NW with respect to the ground. What is the velocity of the plane with respect to the air?

$\vec{V}_{PG} = 80$ m/s @ 30° N of E



$\vec{V}_{AG} = 35$ m/s @ 45° W of N



$\vec{V}_{PA} = \vec{V}_{PG} + \vec{V}_{GA} = \vec{V}_{PG} - \vec{V}_{AG}$

$\vec{V}_{PA} = 80\cos 30\,\hat{i} + 80\sin 30\,\hat{j}$

$= 69.3\hat{i} + 40\hat{j}$

$\vec{V}_{AG} = -35\sin 45\,\hat{i} + 35\cos 45\,\hat{j}$

$= -24.7\hat{i} + 24.7\hat{j}$

So $\vec{V}_{PA} = (69.3\hat{i} + 40\hat{j}) - (-24.7\hat{i} + 24.7\hat{j})$

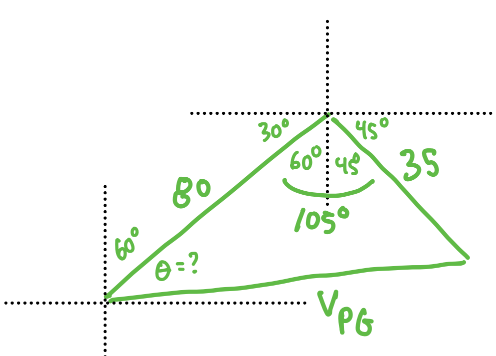
$\boxed{\vec{V}_{PA} = 94\,\hat{i} + 15.3\,\hat{j} \text{ m/s}}$

$v^2 = 94^2 + 15.3^2$

$v = 95.2$ m/s

$\tan\theta = \frac{15.3}{94} = 0.163$

$\theta = 9.2°$



$c^2 = a^2 + b^2 - 2ab\cos C$

$v_{PG}^2 = (80)^2 + (35)^2 - 2(80)(35)\cos(105)$

$= 6400 + 1225 - (-1449.4) = 9074$

$\therefore v_{PG} = 95.3$ m/s

$\frac{\sin\theta}{35} = \frac{\sin 105}{95.3} \rightarrow \sin\theta = 0.355$

$\theta = 20.8° \rightarrow$ lastly $30° - 20.8 = 9.2°$

So $\boxed{\vec{V}_{PG} = 95.3 \text{ m/s} \ @ \ 9.2^{\circ}}$

Relative Motion Problems

Answers:

1. a) 80 mph b) –15 mph c) 25 mph d) 90 mph

2. a) –4 m/s b) 4 m/s c) $v_{AY} = -2.67$ m/s & $v_{BY} = 1.33$ m/s

3) 1.85 m/s

4. a) $r_{AB} = 20t - 1.5t^2$ b) $r_{BA} = -20t + 1.5t^2$ c) 66.7 m d) –20 m/s

5. a) 8***i*** m/s b) –2***i*** m/s c) 3***i*** + 5***j*** m/s d) 6 s e) 18 m
   f) 36.9° W of N (or 126.9°) g) 4 m/s h) 7.5 s i) part c! so 6 seconds

6. a) –40***i*** – 100***j*** m/s (or 108 m/s @ 21.8° W of S) b) –144***i*** – 360***j*** km
   c) 23.6° E of S (or –66.4°) d) 91.7 m/s

7) 94***i*** + 15***j*** m/s